FÓRMULAS MATEMÁTICAS

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1 Fórmulas Útiles

En este apunte encontrará una lista de fórmulas útiles extraídas de [1].

1. Ecuación cuadrática. Si $a \neq 0$, las raíces de $ax^2 + bx + c = 0$ son

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Fórmula del binomio.

$$\begin{aligned}(1+x)^n &= 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n!}{(n-r)!r!}x^r + \ldots \\ &= \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r.\end{aligned}$$

Nota: $0! = 1$. Si n es un entero positivo, la expresión consiste en un número finito de términos. Si n no es un entero positivo, la serie converge para $|x| < 1$ y si $n > 0$, la serie converge también para $|x| = 1$.

3. Fórmula de Stirling

$$\sqrt{2\pi} = \lim_{n\to\infty} \frac{n!}{n^n e^{-n}\sqrt{n}}$$

Este límite permite calcular aproximadamente $n!$ para n grande.

4. Serie geométrica

$$\sum_{k=0}^{n} r^k = 1 + r + r^2 + r^3 + \cdots + r^n = \frac{1-r^n}{1-r}.$$

Si $|r| < 1$,

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}.$$

5. Series

$$\begin{aligned}\sum_{k=1}^{n} k &= 1+2+3+\cdots+n = \frac{n(n+1)}{2}\\ \sum_{k=1}^{n} k^2 &= 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}\\ \sum_{k=1}^{n} k^3 &= 1^3+2^3+3^3+\cdots+n^3 = \frac{n^2(n+1)^2}{4}\end{aligned}$$

6. Serie de Taylor

$$\begin{aligned}f(x+h) &= \sum_{k=0}^{n-1}\frac{h^k}{k!}f^k(x)+R_n\\ &= f(x)+hf(x)+\frac{h^2}{2!}f(x)+\cdots+\frac{h^{n-1}}{(n-1)!}f^{n-1}(x)+R_n,\end{aligned}$$

donde

$$R_n = \frac{h^n}{n!}f^n(x+\theta h),$$

con θ entre 0 y 1.

7. Funciones trigonométricas

- $\mathrm{sen}^2(x)+\cos^2(x)=1$
- $\mathrm{sen}(x\pm y)=\mathrm{sen}(x)\cos(y)\pm\cos(x)\mathrm{sen}(y)$
- $\cos(x\pm y)=\cos(x)\cos(y)\mp\mathrm{sen}(x)\mathrm{sen}(y)$
- $2\mathrm{sen}(x)\cos(y)=\mathrm{sen}(x+y)+\mathrm{sen}(x-y)$
- $2\mathrm{sen}(x)\cos(y)=\cos(x-y)-\cos(x+y)$
- $2\cos(x)\cos(y)=\cos(x+y)+\cos(x-y)$
- $\mathrm{sen}(2x)=2\mathrm{sen}(x)\cos(x)$
- $\cos(2x)=\cos^2(x)-\mathrm{sen}^2(x)$
- $\mathrm{sen}(x)=1/2\,i(e^{ix}-e^{-ix})$
- $\cos(x)=1/2\,(e^{ix}+e^{-ix})$
- $e^{ix}=\cos(x)+i\mathrm{sen}(x)$

8. Operadores diferenciales

El gradiente de una función escalar $\phi(x,y,z)$ da el valor absoluto y la dirección en la que ϕ varía más rápidamente. Es perpendicular a la superficie ϕ =constante. En coordenadas cartesianas,

$$\mathrm{grad}_\phi = \nabla\phi = \frac{\partial\phi}{\partial x}\mathbf{i}+\frac{\partial\phi}{\partial y}\mathbf{j}+\frac{\partial\phi}{\partial z}\mathbf{k}.$$

La divergencia de un vector A en coordenadas cartesianas está dada por,

$$\mathrm{div}\,\mathbf{A} = \nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}.$$

El rotor de un vector $\mathbf{A}$ en coordenadas cartesianas está dado por,

$$\mathrm{rot}\,\mathbf{A} = \nabla\times\mathbf{A} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ A_x & A_y & A_z\end{vmatrix}$$

9. Identidades vectoriales

- $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = (\mathbf{B}\times\mathbf{C})\cdot\mathbf{A} = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) = (\mathbf{C}\times\mathbf{A})\cdot\mathbf{B}$
- $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{C}\times\mathbf{B})\cdot\mathbf{A} = (\mathbf{A}\cdot\mathbf{C})\mathbf{B}-(\mathbf{A}\cdot\mathbf{B})\mathbf{C}$
- $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})+\mathbf{B}\times(\mathbf{C}\times\mathbf{A})+\mathbf{C}\times(\mathbf{A}\times\mathbf{B})=0$
- $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D})-(\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$
- $(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\times\mathbf{B}\cdot\mathbf{D})\mathbf{C}-(\mathbf{A}\times\mathbf{B}\cdot\mathbf{C})\mathbf{D}$
- $\nabla(fg)=\nabla(gf)=f\nabla g+g\nabla f$
- $\nabla\cdot(f\mathbf{A})=f\nabla\cdot\mathbf{A}+\mathbf{A}\cdot\nabla f$
- $\nabla\times(f\mathbf{A})=f\nabla\cdot\mathbf{A}+\mathbf{A}\times\nabla f$

- $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$
- $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$
- $\mathbf{A} \times (\nabla \times \mathbf{B}) = (\nabla \mathbf{B}) \cdot \mathbf{A} - \mathbf{A} \cdot \nabla \mathbf{B}$
- $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$
- $\nabla^2 f = \nabla \cdot \nabla f$
- $\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A})$
- $\nabla \times \nabla f = 0$
- $\nabla \cdot \nabla \times \mathbf{A} = 0$

10. Ecuaciones diferenciales vectoriales

(a) Coordenadas cilíndricas. Divergencia:

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

(b) Coordenadas cilíndricas. Gradiente:

$$(\nabla f)_r = \frac{\partial f}{\partial r}, \quad (\nabla f)_\phi = \frac{1}{r}\frac{\partial f}{\partial \phi}, \quad (\nabla f)_z = \frac{\partial f}{\partial z}$$

(c) Coordenadas cilíndricas. Rotor:

$$\begin{aligned} (\nabla \times \mathbf{A})_r &= \frac{1}{r}\frac{\partial A_r}{\partial \phi} - \frac{\partial A_\phi}{\partial z}, \\ (\nabla \times \mathbf{A})_\phi &= \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}, \\ (\nabla \times \mathbf{A})_z &= \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi} \end{aligned}$$

(d) Coordenadas esféricas. Divergencia:

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\,\mathrm{sen}(\theta)}\frac{\partial}{\partial \theta}(A_\theta \mathrm{sen}(\theta)) + \frac{1}{r\,\mathrm{sen}(\theta)}\frac{\partial A_\phi}{\partial \phi}$$

(e) Coordenadas esféricas. Gradiente:

$$(\nabla f)_r = \frac{\partial f}{\partial r}, \quad (\nabla f)_\theta = \frac{1}{r}\frac{\partial f}{\partial \theta}, \quad (\nabla f)_\phi = \frac{1}{r\,\mathrm{sen}(\theta)}\frac{\partial f}{\partial \phi}$$

(f) Coordenadas esféricas. Rotor:

$$\begin{aligned} (\nabla \times \mathbf{A})_r &= \frac{1}{r\,\mathrm{sen}(\theta)}\frac{\partial}{\partial \theta}(A_\phi \mathrm{sen}(\theta)) - \frac{1}{r\,\mathrm{sen}(\theta)}\frac{\partial A_\theta}{\partial \phi}, \\ (\nabla \times \mathbf{A})_\theta &= \frac{1}{r\,\mathrm{sen}(\theta)}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi), \\ (\nabla \times \mathbf{A})_\phi &= \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta} \end{aligned}$$

Referencias

[1] Anderson H. L., 1981. En Physics Vade Mecum, editado por Anderson H. L., American Institute of Physics.